Measuring & Managing Financial Risks with Improved Alternatives Beyond Value-At-Risk (VaR)

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Why Alternatives to VaR are Needed
What is the Critical Limitation of VaR?


Financial institutions are subject to many sources of risk.

Risk: Degree of uncertainty about future net returns.

Credit risk: Potential loss due to the inability of a counterparty to meet its obligations.
- credit exposure, probability of default and loss in the event of default.

Liquidity risk: If a firm needs some liquidity, it may be compelled to sell highly illiquid assets at a discount.

Market risk: uncertainty of future earnings, due to the changes in market conditions.

Operational risk: errors in settling payments or transactions, includes the risk of fraud and regulatory risks.
**Why Alternatives to VaR are Needed**

**What is the Critical Limitation of VaR?**


**Systemic risk** Risk that the stability of the financial system as a whole is threatened: a single institution’s risk measure does not necessarily reflect systemic risk (Adrian & Brunnermeier, 2011). *

**Systemic risk measures** capture the potential for the spreading of financial distress across institutions by gauging this increase in tail co-movement.

**Systemic risk** “Risk that an event will trigger a loss of economic value or confidence in, and attendant increases in uncertainty about substantial portion of the financial system that is serious enough to quite probably have significant adverse effects on the real economy” (ESRB, 2009).

“The risk that institutional distress spreads widely and distorts the supply of credit and capital to the real economy” (Adrian & Brunnermeier 2009).

“[The risk] of widespread failures of financial institutions or freezing up of capital markets that can substantially reduce the supply of such intermediated capital to the real economy” (Acharya et al. 2009).
Why Alternatives to VaR are Needed

Basel 2.5: Proposed and Implemented Risk Measures


Early Warning Joint Indicators for Asset Bubbles: asset price misalignments based upon credit, equity prices and property prices, global financial variables (ESRB 2009, Borio & Drehmann 2009, Alessi & Detken 2009)


Source: Bank capital: Half-cocked Basel: Stop-gap rules on banks’ trading books may add perilous complexity, The Economist, Jan 7th 2012. The risk of a trading portfolio must now be broken down into five “buckets” — Value at risk (VaR), a measure of how much could be lost in an average trading day; Stressed VaR (how much could be lost in extreme conditions); Plus three types of credit risk ranging from the risk of single credits to those of securitised loans.
Why Alternatives to VaR are Needed
In 2001... Did we Know About ‘2008’?


• Proposed regulations fail to consider the fact that risk is endogenous. Value-at-Risk can destabilise an economy and induce crashes when they would not otherwise occur.

• Statistical models used for forecasting risk have been proven to give inconsistent and biased forecasts, notably under-estimating the joint downside risk of different assets. The Basel Committee has chosen poor quality measures of risk when better risk measures are available.

• Heavy reliance on credit rating agencies for the standard approach to credit risk is misguided as they have been shown to provide conflicting and inconsistent forecasts of individual clients' creditworthiness. They are unregulated and the quality of their risk estimates is largely unobservable.

• Financial regulation is inherently procyclical. Our view is that this set of proposals will, overall, exacerbate this tendency significantly. In so far as the purpose of financial regulation is to reduce the likelihood of systemic crisis, these proposals will actually tend to negate, not promote this useful purpose.
Why Alternatives to VaR are Needed
In 2001... Did we Know About ‘2008’?


“Firstly, existing risk models treat risk as a fixed exogenous process. This, however, is not the case. Market volatility is, in part at least, the outcome of interaction between market players and is thus endogenous. This endogeneity may matter enormously in times of crisis. By failing to recognise it, existing models produce inaccurate risk predictions and it is not clear how this systemic dimension of risk is to be treated in the proposals.”

“Secondly, VaR is a misleading risk measure when the returns are not normally distributed, as is the case with credit, market and, in particular, operational risk. Moreover, it does not measure the distribution or extent of risk in the tail, but only provides an estimate of a particular point in the distribution. Existing VaR models generate imprecise and widely fluctuating risk forecasts.”
“However, this reasoning is faulty. Volatility is determined in the market, in large part by the behaviour of all individual market participants - in other words, risk is endogenous by definition. The failure to recognise this endogeneity is relatively innocuous during times of ‘calm’ in which the actions of many heterogeneous market participants (in terms of risk-aversion, portfolio positions etc.) more or less cancel each other out.”

“In times of crisis, in contrast, this endogeneity may matter enormously if agents become more homogeneous as a result. Using similar risk models, they may pursue similar strategies to mitigate the adverse effects of the on-setting crisis. In such a case, individual actions do not ‘more or less cancel each other out’ but may in fact reinforce each other. More importantly, one has to wonder about the impact of regulation on the endogeneity of risk and liquidity... ”
“For example, is it the case that [VaR] regulation renders market players more homogenous and thus aggravates the instability of banking systems? In times of crisis, in contrast, this endogeneity may matter enormously if agents become more homogeneous as a result. In such a case, individual actions do not ‘more or less cancel each other out’ but may in fact reinforce each other.”

“Furthermore, the mechanism just described may trigger a market collapse that would not occur if VaR regulation were not present. When, for example, prices fall banks must sell risky assets to fulfil their binding regulatory constraints. In the absence of regulation, less risk-averse banks would be able and willing to provide liquidity by buying these assets. In a regulated economy, however, regulatory constraints restrict their ability to do so. Eventually, markets for such assets break down. Such a breakdown would not occur if VaR regulation were absent.”
Problem of appropriate **risk measurement time horizon**: Is 10 days ok?

**Scaling of short-horizon VaR** to longer time horizon with common square-root-of-time scaling rule found inaccurate.

**Time-varying volatility** is a feature of many financial time series and can have important ramifications for VaR measurement.

VaR without time-varying volatility can **dangerously under-estimate risk**, when true underlying risk factors exhibit time-varying volatility.

**Extreme liquidity risk** wherein **collective liquidation** of positions occurs is not accounted for in existing VaR Measures.
Improved Alternatives beyond VaR

Need for Improvements Beyond VaR


VaR *lacks* subadditivity i.e., its compartmentalised risk measurement based is not necessarily conservative.

The most prominent alternative to VaR is *expected shortfall*, which is *subadditive* (Basel Committee 2011).

*Spectral risk measures* are a promising generalisation of expected shortfall.

VaR doesn’t factor in *stress testing* and “*stressed VaR*” approach has not been adequately studied or analyzed.

Need to move to unified or *integrated risk measurement* that considers all risks jointly to factor in *compounding effects*.

VaR capital requirements are of *procylical nature* and induce cyclical behaviors that exacerbate the economic cycle.
Coherent risk measure denotes the amount of cash that has to be added to a portfolio to make its risk acceptable (Hull 2009).

- The risk measures for two portfolios after they have been merged should be no greater than the sum of their risk measures before they were merged.

**VaR is not a coherent risk measure because it may violate the subadditivity criterion which reflects the idea that risk can be reduced by diversification.**

- If a regulator uses a non-subadditive risk measure in determining the regulatory capital for a financial institution, that institution has an incentive to legally break up into various subsidiaries in order to reduce its regulatory capital requirements.
A Risk Measure is “coherent” if it satisfies all of the following four axioms (Atzner et al. 1999):

Subadditivity (diversification) \( R(L_1 + L_2) \leq R(L_1) + R(L_2) \)

Positive homogeneity (scaling) \( R(\lambda L) = \lambda R(L) \), for every \( \lambda > 0 \)

Monotonicity \( R(L_1) < R(L_2) \) if \( L_1 < L_2 \)

Transition property \( R(L + a) < R(L) - a \)
ES is most well-known risk measure following VaR. Conceptually intuitive and firm theoretical background

Preferred to VaR by an increasing number of FIs and risk managers.

**ES corrects three shortcomings of VaR.**

1. Accounts for the severity of losses beyond confidence threshold.
   - Especially important for regulators.
2. Always subadditive and coherent.
3. Mitigates impact that choice of specific confidence level may have on risk management decisions,
   - Particularly given seldom an objective reason for this choice.
VaR vs. ES (Conditional VaR)

<table>
<thead>
<tr>
<th>VaR</th>
<th>Expected Shortfall (Conditional VaR)</th>
</tr>
</thead>
<tbody>
<tr>
<td>&quot;How bad can things go&quot;</td>
<td>“If things do go bad, what is expected loss”</td>
</tr>
</tbody>
</table>

X % certain that we will not lose more than SV in Time T. Typical X% = 1%, 5%; Typical T = 1 day, 2 week.

Expected loss during time T, conditional on loss being greater than Xth %ile of loss distribution.

(Hull, 2009)
Surveys & Current State of Risk Measures: Expected Shortfall


To define ES, let $L$ be a random loss with distribution function $F_L$ and $\alpha \in (0,1)$ a confidence level (close to 1). Recall that the $\alpha$-VaR is defined as the $\alpha$-quantile of $F_L$. The ES at level $\alpha$ is defined by

$$ES_\alpha = \frac{1}{1-\alpha} \int_\alpha^1 \text{VaR}_u(L) \, du$$

and can thus be understood as an average of all VaRs from level $\alpha$ up to 1. ES is a coherent risk measure – and so subadditive. It is continuous in $\alpha$ and thus avoids cliff effects that may appear when the distribution has discrete components.

If the loss distribution is continuous, there is an even more intuitive representation:

$$ES_\alpha = E(L|L \geq \text{VaR}_\alpha)$$

i.e. ES is then the expected loss conditional on this loss belonging to the $100(1-\alpha)$ percent worst losses.
“VaR was never meant to be a tool for regulating banks. We believe that lack of systemic risk measure is at the root of practical failures of regulation... The gap between the theoretical recommendations and the practical needs of regulators has been so wide that measures such as institution-level VaR have persisted in assessing risks of the financial system as a whole.” (Acharya et al. 2010)

Current regulation and measurement is aimed at limiting each institution’s risk in isolation without enough attention to systemic risk (Acharya et al. 2009).

Marginal expected shortfall (MES) and Systemic expected shortfall (SES) measures modeled on recent financial crisis based upon analysis of 102 financial firms in US. Calculated MES of each firm using worst 5% days of the value-weighted market return from CRSP. Checked how well these risk measures calculated before sub-prime crisis help predict which institutions fared the worst during the crisis (July 2007 - December 2008). Measures explain significant proportion of realized returns during crisis ($R^2$ of 27.34%). In contrast, institutional measure of expected loss in institution’s own left tail (ES) does a poor job.
Marginal expected shortfall (MES): Measures loss in case returns go below certain %ile of distribution (i.e. 1% or 5% on left side). How each group’s risk taking adds to the financial institution’s overall risk. Can also be calculated for financial institution as a whole: Contribution of each FI to risk of complete financial system.

Systemic expected shortfall (SES): An FI’s propensity to be undercapitalized when the system as a whole is undercapitalized, which increases in its leverage, volatility, correlation, and tail-dependence (Acharya et al. 2010). Related to MES taking leverage and risk taking into account. Measures externalities when aggregate banking capital drops below a certain threshold: increases for high leverage & risks.

With use of SES and MES, banks have incentive to reduce tax (or insurance) payments and take into account externalities arising from their risks and default.

Limitations of MES & SES: Difficult to determine when systemically relevant institutions are likely to fail and cause spillovers to the real economy (Eijffinger, 2009).
Marginal expected shortfall (MES):

\[ q_\alpha = \sup \{ z \mid Pr [R < r] \leq \alpha \} \]

\[ ES_\alpha = -E [R \mid R \leq q_\alpha] \]

\[ ES_\alpha = -\sum_i y_i E [r_i | R \leq q_\alpha] \]

\[ \frac{\partial ES_\alpha}{\partial y_i} = -E [r_i | R \leq q_\alpha] \equiv MES^i_\alpha \]

- Measures how group i's risk taking adds to the bank's overall risk.
- Can be measured by estimating group i's losses when the firm as a whole is doing poorly.

Systemic expected shortfall (SES):

Default Expected Shortfall:

\[ DES^i \equiv -E \left[ I_i \cdot w^i_1 \right] \]

Further, we define bank i's systemic expected shortfall (SES) as its the amount its equity \( w^i_1 \) drops below its target level, which is a fraction \( z \) of assets \( a^i \) in case of a systemic crisis:

\[ SES^i \equiv E \left[ \bar{I} \cdot (za^i - w^i_1) \right] \]
“Co-risk management” tool CoVaR$_i$: Whole system’s (i.e., portfolio’s) VaRs conditioned on institution $i$ being in distress, i.e., being at its unconditional VaR$_i$ level.

Use CoVaR to calculate marginal contribution of institution $i$ to the overall systemic risk

$\Delta$CoVaR$_i$ : Difference between CoVaR and the unconditional whole system’s VaR. Determines how much an institution adds to overall systemic risk.

**Definition 1** We denote by CoVar$_q^{j|i}$ the VaR of institution $j$ (or the financial system) conditional on some event $C(X^i)$ of institution $i$. That is, CoVar$_q^{j|i}$ is implicitly defined by the $q$-quantile of the conditional probability distribution:

$$
Pr\left(X^i \leq \text{CoVar}_q^{j|i}(X^i) \mid C(X^i)\right) = q.
$$

We denote institution $i$’s contribution to $j$ by

$$
\Delta\text{CoVar}_q^{j|i} = \text{CoVar}_q^{j|i}(X^i=\text{VaR}_q^j) - \text{CoVar}_q^{j|i}(X^i=\text{Median}^i).
$$
Surveys & Current State of Risk Measures: Co-VaR (Co-Value-At-Risk) Handicapped

Source: Rodríguez-Moreno, M. and Ignacio, J. (2010). Systemic risk measures: the simpler the better?. BIS (Bank for International Settlements) Papers No 60. 29-35..

Comparison of six systemic risk measures based on (i) principal components of the bank's credit default swaps (CDS); (ii) interbank interest rates; (iii) structural credit risk models; (iv) collateralised debt obligation (CDO) indices and their tranches; (v) multivariate densities computed from CDS spreads; and (vi) co-risk measures: CoVaR.

All measures benchmarked using: Granger Causality tests; Gonzalo and Granger metric; and McFadden R-squared

CoVaR found to be least reliable measure of systemic risks.

Conclusion:

Best-performing measures of systemic risk are based on simple indicators obtained from credit derivatives and interbank rates.

Indicators relying on complex statistical procedures or questionable assumptions such as CoVaR do not perform particularly well.

Implications for investors and regulators: look for simple, robust indicators based directly on liquid market prices of credit-sensitive instruments; beware of overcomplicated modelling based on dubious assumptions.
## Surveys & Current State of Risk Measures: Co-VaR (Co-Value-At-Risk) Handicapped

*Source: Rodríguez-Moreno, M. and Ignacio (2010), J. Systemic risk measures: the simpler the better?. BIS (Bank for International Settlements) Papers No 60. 29-35.*

### Panel A: European Portfolio

<table>
<thead>
<tr>
<th>Causality Test</th>
<th>Price Discovery</th>
<th>McFadden R-squared</th>
<th>Final Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>PCA</td>
<td>2</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>LO</td>
<td>2</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>SIN05</td>
<td>-1</td>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td>CDO</td>
<td>1</td>
<td>2</td>
<td>-5</td>
</tr>
<tr>
<td>BSI</td>
<td>0</td>
<td>-3</td>
<td>3</td>
</tr>
<tr>
<td>ΔCoES</td>
<td>-4</td>
<td>-5</td>
<td>-3</td>
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</table>

### Panel B: US Portfolio

<table>
<thead>
<tr>
<th>Causality Test</th>
<th>Price Discovery</th>
<th>McFadden R-squared</th>
<th>Final Score</th>
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</thead>
<tbody>
<tr>
<td>PCA</td>
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<td>5</td>
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<tr>
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<td>SIV10</td>
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<tr>
<td>CDO</td>
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<td>-6</td>
</tr>
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<td>BSI</td>
<td>-3</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>ΔCoES</td>
<td>-3</td>
<td>-2</td>
<td>2</td>
</tr>
<tr>
<td>ΔCoVaR</td>
<td>0</td>
<td>-6</td>
<td>0</td>
</tr>
</tbody>
</table>
Appendix: Quantitative Models of Risk Measures
Surveys & Current State of Risk Measures


**VaR** (p. 17)

\[ \text{VaR}_\alpha (L) = \inf \{ l : F_L (l) \geq \alpha \} \]

**Expected shortfall** (p. 21)

\[ \text{ES}_\alpha = \frac{1}{1 - \alpha} \int_\alpha^1 \text{VaR}_u (L) \, du \]

\[ \text{ES}_\alpha = E \left( L \mid L \geq \text{VaR}_\alpha \right) \]
Spectral risk measures (p. 23) \[ SRM = \int_0^1 w(u) \text{VaR}_u(L) du \]

Expected shortfall, special case of SRM where:

\[ w(u) = (1 - \alpha)^{-1} 1_{\{0 \leq u \leq 1\}} \]
Appendix: Quantitative Models of Risk Measures
Surveys & Current State of Risk Measures


Distortion risk measures (p. 24)

Each SRM is a DM as

\[ D(u) = \int_0^u w(s) \, ds \]

VaR is represented by DM where

\[ D_{\text{VaR}}(u) = 1_{\{u \geq \alpha\}} \]
Upper partial moments (p. 25)

Relation with Expected Shortfall

\[ UPM(k, q) = \int_q^\infty (1 - q)^k dF_L(l) \]

\[ UPM(1, \text{VaR}_\alpha) = (1 - \alpha)(\text{ES}_\alpha - \text{VaR}_\alpha) \]

Left-tail measure (p. 25)

\[ LTM = \sqrt{E\left[ (L - E(L|L \geq \text{VAR}_\alpha))^2 \Big| L \geq \text{VAR}_\alpha \right]} \]
Suppose that we observe a vector of portfolio returns, \( \{y_t\}_{t=1}^T \). Let \( \theta \) be the probability associated with VaR, let \( x_t \) be a vector of time \( t \) observable variables, and let \( \beta_\theta \) be a \( p \)-vector of unknown parameters. Finally, let \( f_t(\beta) \equiv f_t(x_{t-1}, \beta_\theta) \) denote the time \( t \) \( \theta \)-quantile of the distribution of portfolio returns formed at time \( t - 1 \), where we suppress the \( \theta \) subscript from \( \beta_\theta \) for notational convenience. A generic CAViaR specification might be the following:

\[
f_t(\beta) = \beta_0 + \sum_{i=1}^{q} \beta_i f_{t-i}(\beta) + \sum_{j=1}^{r} \beta_j l(x_{t-j})
\]
References: Alternative Measures Beyond VaR


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  Other versions:
References: Alternative Measures Beyond VaR


• Rodríguez-Moreno, M. and Ignacio (2010), J. Systemic risk measures: the simpler the better?. BIS (Bank for International Settlements) Papers No 60. 29-35.